

Name

Current school



WELLINGTON COLLEGE

13+ SCHOLARSHIP EXAMINATION 2022

MATHEMATICS

TIME ALLOWED: 90 minutes

TOTAL MARKS: 90 (Marks for each question are shown in square brackets)

This paper is divided into two sections:

Section A is worth 30 marks and contains seven questions. You should attempt all questions in Section A.

Section B is worth 60 marks and contains six questions each worth 10 marks. You may attempt all questions. Start with the ones that interest you most; answer as many questions as you can. You may find some easier than others.

Write your answers on the question paper.

You may use a calculator.

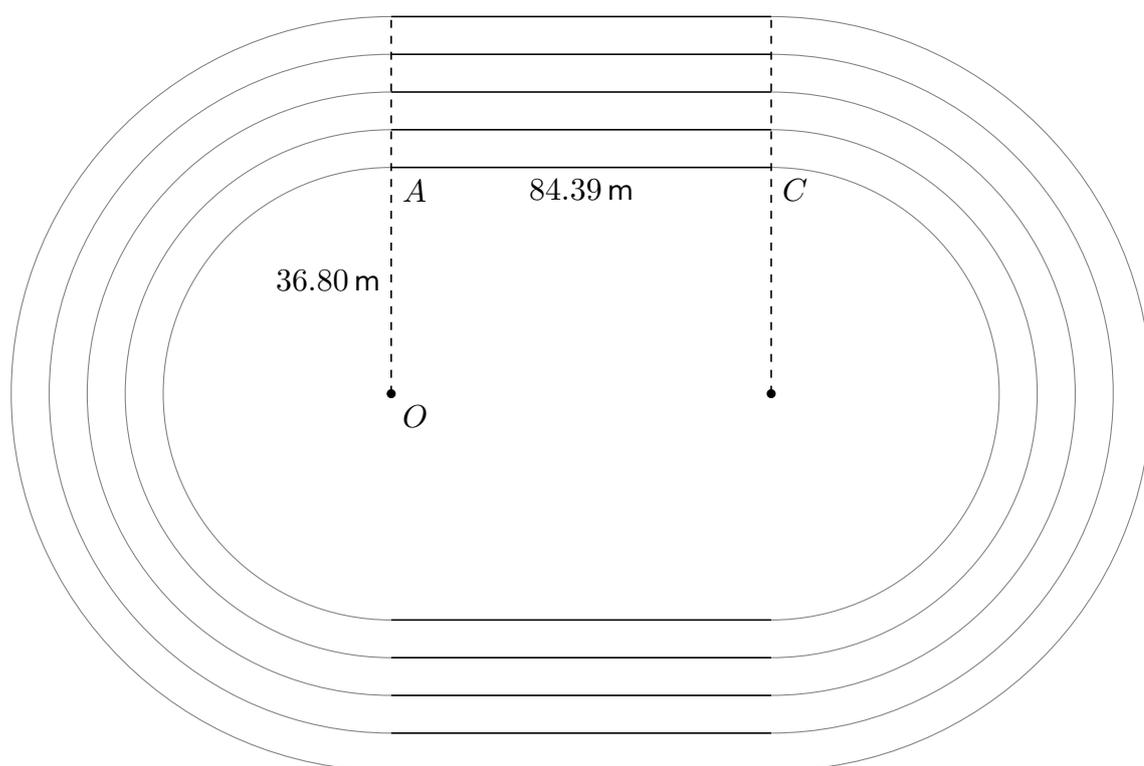
Credit will be given for the clarity of your work and your explanations.

Section B (60 marks)

8. An running track with four lanes is designed so that the inside of the track (the inside edge of the inside lane) is 400 m long.

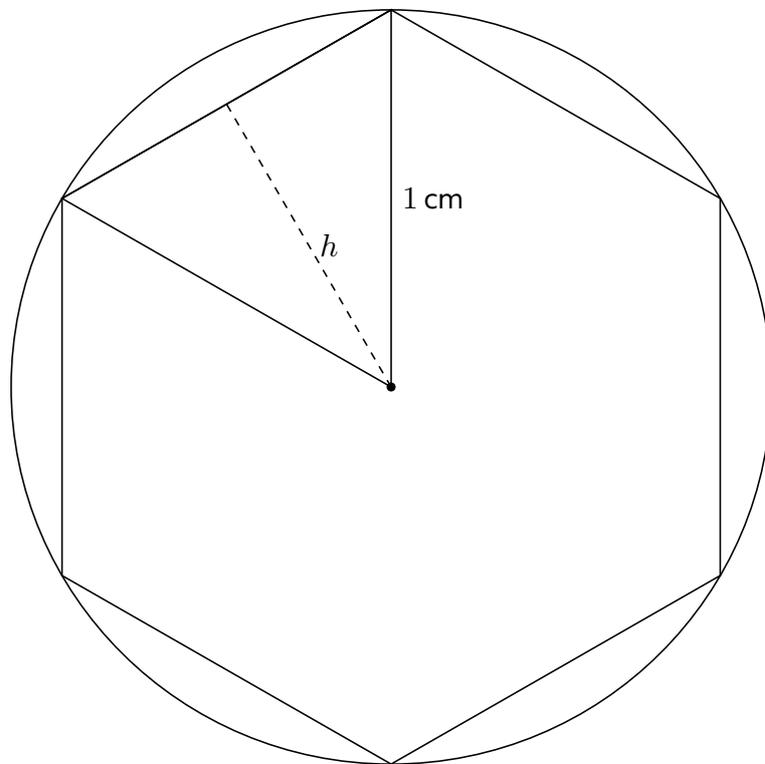
The diagram below shows the track.

Each lane is 122 cm wide. The straight sides (length AC below) are 84.39 m long and the radius (length OA below) of the inside lane is 36.80 m.



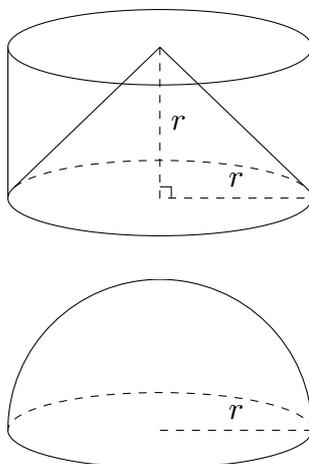
- (a) Show that the inside lane is indeed the correct length, to one decimal place. [3]
- (b) Find the length of the outside of the track. [2]
- (c) Runner A runs 400 m in one minute. Runner B runs along the outside line of the track in the same time, by what percentage is their speed greater than runner A. [2]
- (d) The track surface is 13.5 mm thick and has a density of 933 kg/m^3 , how many tonnes of material are needed for the track surface. The empty space in middle of the track should not be counted here. [3]

9. The diagram below shows a regular hexagon inscribed within a circle of radius 1 cm. This means that the vertices of the hexagon are on the circle.



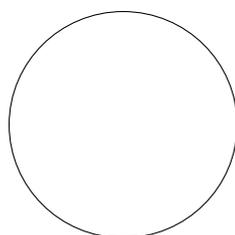
- (a) What is the side length of the hexagon? [1]
- (b) The hexagon can be split into triangles as shown above, find h . [1]
- (c) What is the area of the hexagon as a percentage of the area of the circle? [3]

12. The diagram below shows a hemisphere of radius r and a cone contained within a cylinder, both with base radius r and height r .



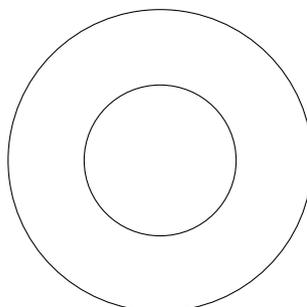
- (a) Show that the volume of the hemisphere is the same as the volume of the cylinder with the cone cut out of it. [2]

The top of the sphere is sliced off horizontally h units (where $0 < h < r$) above its circular base. The top of the shape now formed is a circle:



- (b) What is the radius of this circle, in terms of h ? [2]

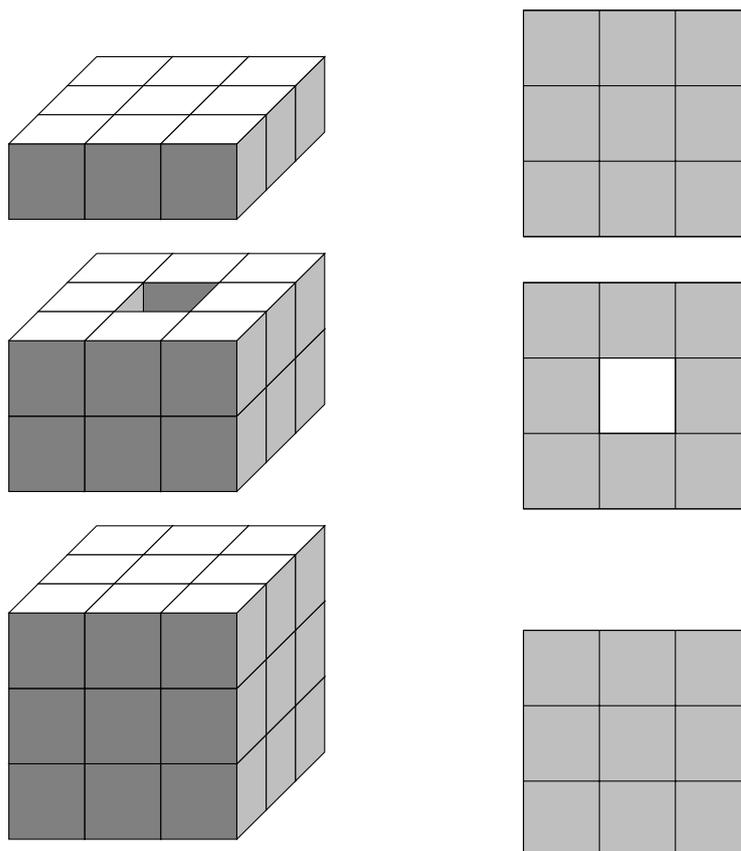
Similarly, the top of the other shape is removed, this time k units below its top, to leave an annulus:



13. In the final part of this question, marks will be awarded for a good approach to solving the problem as well as for the final answer itself. You are encouraged to show your working and findings clearly.

I have twenty six small cubes, each having side length 1 cm. I glue them together to make a larger, hollow cube. Wherever two faces touch I glue the faces together.

The images below show the structure of the cube, one layer at a time. The left images are a 3D view and the right are top down views of the three layers.



- (a) What are the dimensions of the outside of the cube and what are the dimensions of the hollow central void? [1]
- (b) State the side length of the cube shaped void when a large hollow cube of outside dimensions n cm is made. (Assume that $n > 2$) [1]
- (c) Now give a formula for the number of small cubes needed to make such a hollow cube. [1]
- (d) How many pairs of faces would be glued together when I make the large hollow cube using 26 small cubes? [1]

