Name ....

Current school ....

WELLINGTON

COLLEGE

13+ SCHOLARSHIP EXAMINATION 2021

MATHEMATICS

TIME ALLOWED: 90 minutes

You may use a calculator.

The marks available for each question are shown in square brackets.

This paper is divided into two sections:

Section A is worth 30 marks and contains seven questions. You should attempt all questions in Section A.

Section B is worth 60 marks and contains six questions each worth 10 marks. You may attempt all questions. Start with the ones that interest you most; answer as many questions as you can. You may find some easier than others.

Write your answers on the question paper.

Credit will be given for the clarity of your work and your explanations.

Section A (30 marks)

1. Solve

   (a) $4x + 2 = 14$ [1]

   (b) $2x + 5 = 5x - 40$ [1]

   (c) $\frac{x}{5} + \frac{x}{6} = 22$ [2]

2. (a) Express the ratio 240 : 300 : 330 in its simplest form. [1]

(b) Billy, Millie and Tilly share a prize of £852 in the ratio 2 : 3 : 7. Determine the amount each receives. [2]

(c) Fred, Jed and Ved share a prize in the ratio 4 : 3 : 5. Ved receives £82 more than Jed. Determine the amount that Fred receives. [2]

3. Expand and simplify

(a) $5(x-1)+6(3x-1)$ [1]

(b) $2(5x-1)-2(3x-4)$ [1]

(c) $(x-5)(x+3)$ [1]

(d) $(3x-2)^2$ [2]

4. Make x the subject of

(a) $y = 4x - 8$ [1]

(b) $3x + y - 2 = 5x - 3y + 6$ [1]

(c) $\dfrac{3}{2x - 1} = y - 2$ [2]

5. Factorise fully

(a) $14x - 35$ [1]

(b) $6a^2 - 4a$ [1]

(c) $a^2b - ab^2$ [1]

(d) $a^3b - ab^3$ [2]

6. (a) An equilateral triangle has perimeter 180 cm. Write down its side length. [1]

(b) A circle has area 10 cm^2. Find its radius. [2]

7. (a) Increase 80 by 14%. [1]

(b) Decrease 140 by 8%. [1]

(c) 14% of a number is 84. What was the original amount? [2]

Section B (60 marks)

8. I define a new operation, $a \star b$.
   It has the property that for any two numbers x and y,

$$x \star y = xy - x + y.$$

   (a) Calculate $3 \star 4$. [1]

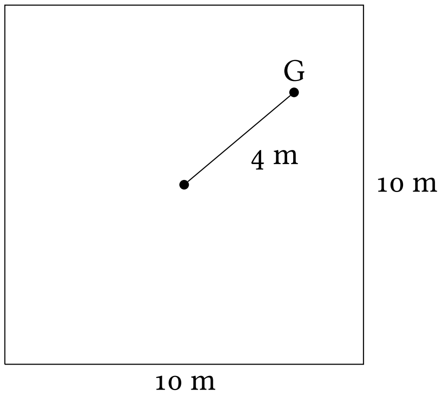
   (b) Show that $a \star 1$ always has the same value, regardless of the value of a. [2]

   (c) Simplify $\frac{1}{2}(b \star a - a \star b)$. [2]

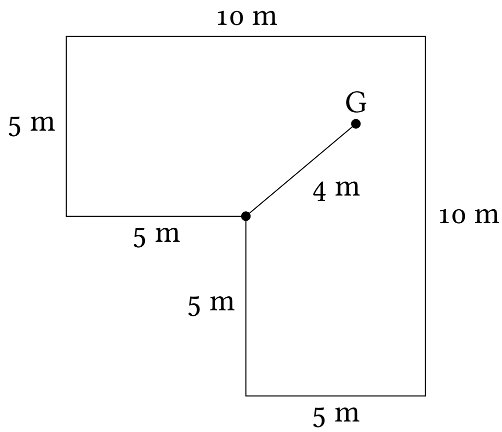
   (d) Solve $4 \star (2 \star x) = 31$. [5]

9. Give your answers correct to 1 decimal place.

(a) A goat is tethered to a rope of length 4 m. The other end of the rope is attached to a pole in the centre of a square grassy field of side length 10 m. [3]
Assuming that the goat can graze anywhere up to 4 m away from the post, find the percentage of the field which the goat can graze.

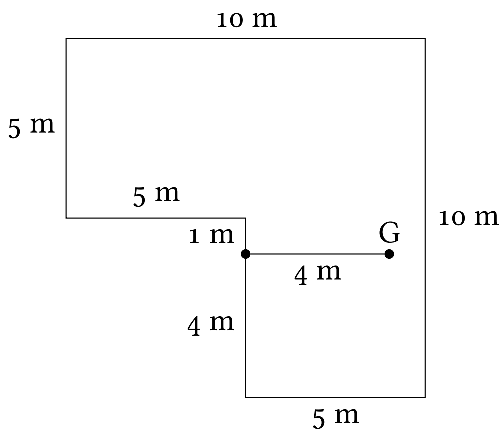


(b) The goat is moved to an L-shaped field, which wraps around a building. It is tethered by a 4 m rope to a fixed point on the corner of the building. [2]



Find the percentage of the field which the goat can graze.

(c) The goat is tethered to a new point in the same field, 1 m away from the corner of the building, as illustrated. [5]



Find the percentage of the field that the goat can graze.

10. In this question, you should use the fact that $24\,682\,468^2 = 609\,224\,226\,571\,024$.

(a) Write down $246\,824\,680^2$. [1]

(b) Find $12\,341\,234^2$. [2]

(c) Find the last four digits of $24\,682\,468^3$. [2]

(d) Expand $(n+2)^2$. [1]

(e) Hence find the central three digits of $24\,682\,470^2$. [4]

11. (a) Rose, Violet and Pearl are all meerkats in a zoo. [5]
When Rose and Violet are weighed together, their combined weight is 10 kg.
When Violet and Pearl are weighed together, their combined weight is 8 kg.
When Pearl and Rose are weighed together, their total weight is 11 kg.
Determine the weight, in kg, of each of the meerkats.

(b) Helga and Horace are two hippos. They are both weighed in January, and their keeper feels that they are underweight. Their feeding pattern is changed and they are weighed again in July. Both have gained weight. [5]
Helga has gained 80 kg.
Horace has gained 90 kg.
Originally, Horace weighed 180 kg more than Helga.
In July, their average weight is 1535 kg.
Find Horace and Helga's January weights.

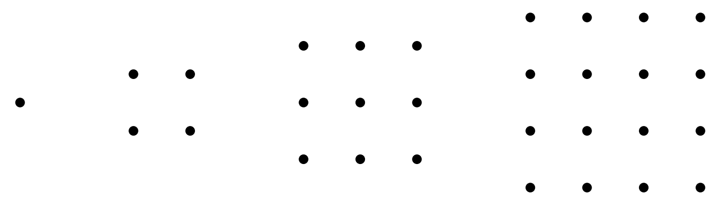
12. (a) Expand (i.e. multiply out) $(A-2)(B-3)$. [1]

(b) Two positive integers, x and y, are such that $xy = 91$. [2]
List all possible pairs of values of x and y.

(c) Two positive integers, p and q, are such that $pq - 2p - 3q = 85$. Use your answers to (a) and (b) to find all possible pairs of values of p and q. [3]

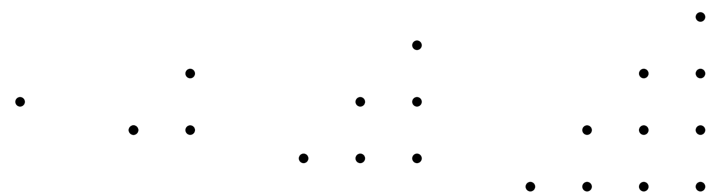
(d) Two positive integers, m and n, are such that $mn - 4m - 5n = 6$. Use a similar approach to find all possible pairs of values of m and n. [4]

13. The square numbers $S_1, S_2, S_3, \ldots$ are so-called because they represent the number of dots required to make a square of varying side lengths:



$S_1 = 1$ $\quad S_2 = 4$ $\quad S_3 = 9$ $\quad S_4 = 16$

The nth square number can be found using the formula $S_n = n^2$.
Similarly, the triangular numbers, $T_1, T_2, T_3, T_4, \ldots$ are so-called because they represent the number of dots required to make triangles of varying side lengths:



$T_1 = 1$ $\quad T_2 = 3$ $\quad T_3 = 6$ $\quad T_4 = 10$

(a) Determine, by means of a diagram, the value of the 5th triangular number, T_5. [1]

(b) Blaise suggests that the nth triangular number can be found by the formula [1]

$$T_n = \frac{n(n+1)}{2}$$

Verify that this formula correctly calculates the 5th triangular number.

The nth tetrahedral number, U_n is formed by adding together the first n triangular numbers.
So $U_1 = 1, U_2 = 1 + 3 = 4, U_3 = 1 + 3 + 6 = 10, U_4 = 1 + 3 + 6 + 10 = 20$ and so on.

(c) Find the value of U_5. [1]

(d) Pierre suggests that the tetrahedral numbers can be found using the formula [2]

$$U_n = \tfrac{3}{2}(n^2 - n) + 1$$

Show clearly that this correctly gives the value of U_1, U_2 and U_3.

(e) Show that it does not correctly give the value of U_4. [1]

Sophie suggests that the tetrahedral numbers can in fact be found using a formula which looks like $U_n = an^3 + bn^2 + cn + d$. Suppose she is correct.

(f) Use the value of U_1 to show that $a + b + c + d = 1$. [1]

(g) Similarly, explain why $8a + 4b + 2c + d = 4$. [1]

(h) Write down two further equations which must be satisfied by a, b, c, and d. You do not need to solve your equations. [2]