Name ....

Current school ....

WELLINGTON

COLLEGE

13+ SCHOLARSHIP EXAMINATION 2022

MATHEMATICS

TIME ALLOWED: 90 minutes

TOTAL MARKS: 90 (Marks for each question are shown in square brackets)

This paper is divided into two sections:

Section A is worth 30 marks and contains seven questions. You should attempt all questions in Section A.

Section B is worth 60 marks and contains six questions each worth 10 marks. You may attempt all questions. Start with the ones that interest you most; answer as many questions as you can. You may find some easier than others.

Write your answers on the question paper.

You may use a calculator.

Credit will be given for the clarity of your work and your explanations.

Section A (30 marks)

1. Expand and simplify

   (a) $3(7x-1)+\frac{5}{2}(6x+16)$ [1]

   (b) $(x-7)(3+3x)$ [1]

   (c) $(2x-7)^2$ [1]

   (d) $(x-1)^3$ [2]

2. Solve

   (a) $3x - 1 = 12$ [1]

   (b) $7x + 3 = 2 - 5x$ [1]

   (c) $\dfrac{x}{5} + \dfrac{x}{4} = 18$ [2]

3. (a) Express the ratio $68 : 85 : 170$ in its simplest form. [1]

(b) Tim, Jim, and Kim share a 1 kg cake in the ratio $2 : 7 : 1$, how many grams of cake do they each eat? [1]

(c) Dom, Rom, and Tom share a prize in the ratio $7 : 6 : 5$. Tom receives $£36$ less than Dom, how much dœs Rom receive? [2]

4. Factorize fully

(a) $7x - 14$ [1]

(b) $xyz^2 - xy^2z$ [1]

(c) $y^4z^2 - y^2z^4$ [2]

5. Make x the subject of

(a) $y = 17x - 5$ [1]

(b) $4x + 3y - 2 = \dfrac{x+y}{2}$ [2]

(c) $\dfrac{4}{x+y} = y + 2$ [3]

6. (a) Increase 70 by 5%. [1]

   (b) Decrease 220 by 7%. [1]

   (c) 17% of a number is 136, what was the original number?. [1]

7. (a) A regular octagon has perimeter 240 cm. How long are its sides? [1]

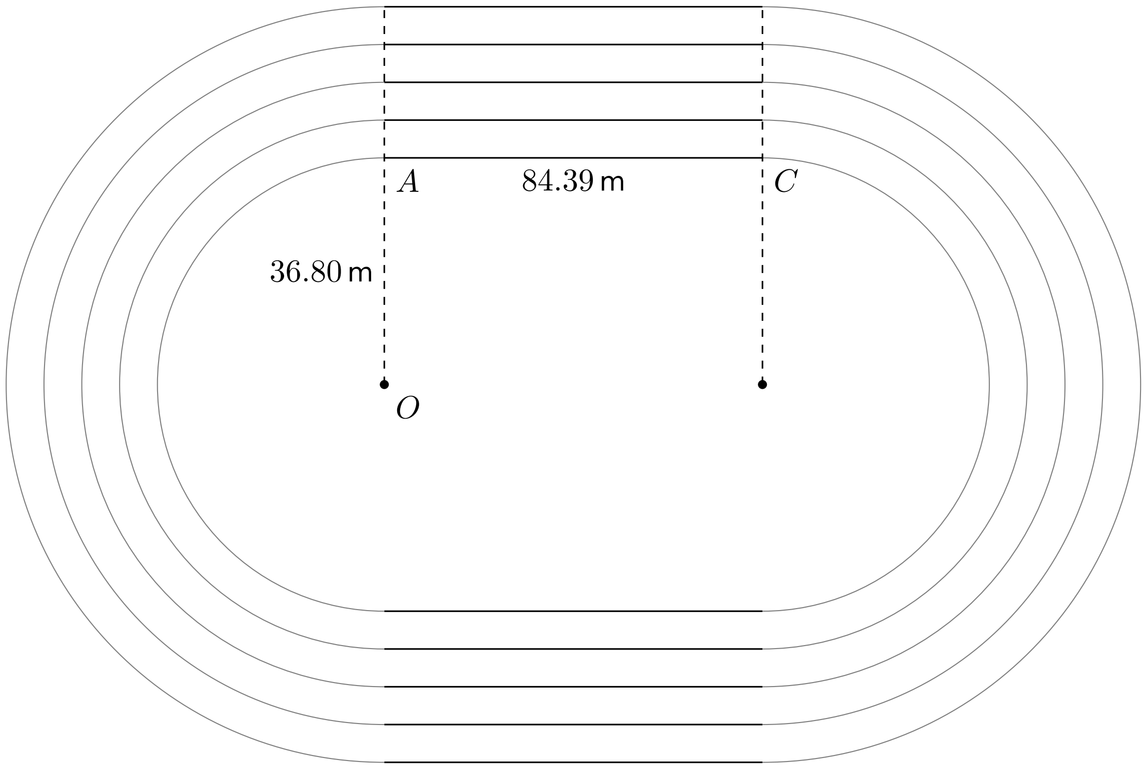
   (b) A quarter of a circle has area $15\,\text{cm}^2$, what is the radius of the full circle? [3]

Section B (60 marks)

8. An running track with four lanes is designed so that the inside of the track (the inside edge of the inside lane) is 400 m long.

   The diagram below shows the track.

   Each lane is 122 cm wide. The straight sides (length AC below) are 84.39 m long and the radius (length OA below) of the inside lane is 36.80 m.



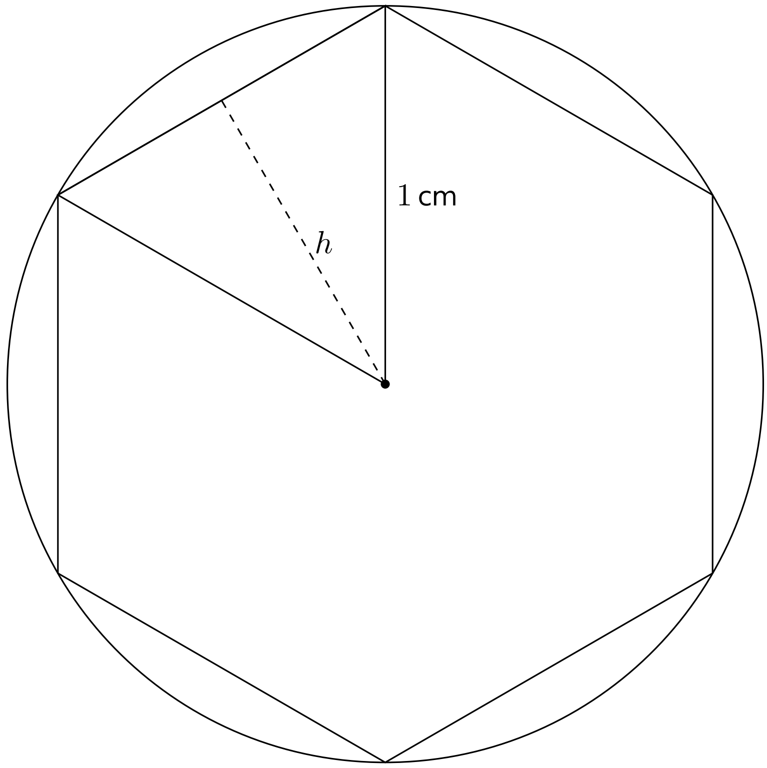
   (a) Show that the inside lane is indeed the correct length, to one decimal place. [3]

   (b) Find the length of the outside of the track. [2]

   (c) Runner A runs 400 m in one minute. Runner B runs along the outside line of the track in the same time, by what percentage is their speed greater than runner A. [2]

   (d) The track surface is 13.5 mm thick and has a density of 933 kg/m^3, how many tonnes of material are needed for the track surface. The empty space in middle of the track should not be counted here. [3]

9. The diagram below shows a regular hexagon inscribed within a circle of radius 1 cm. This means that the vertices of the hexagon are on the circle.

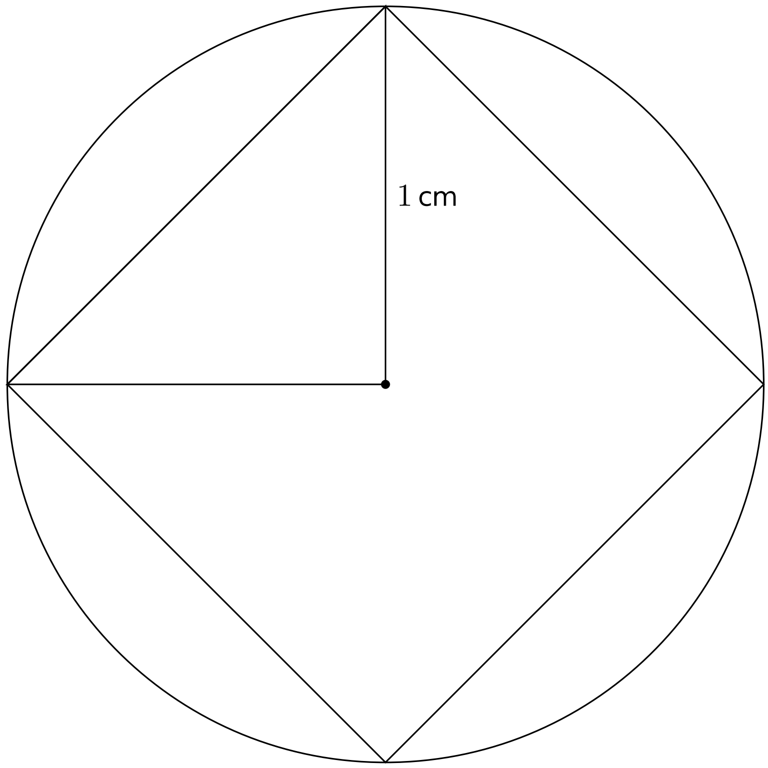


(a) What is the side length of the hexagon? [1]

(b) The hexagon can be split into triangles as shown above, find h. [1]

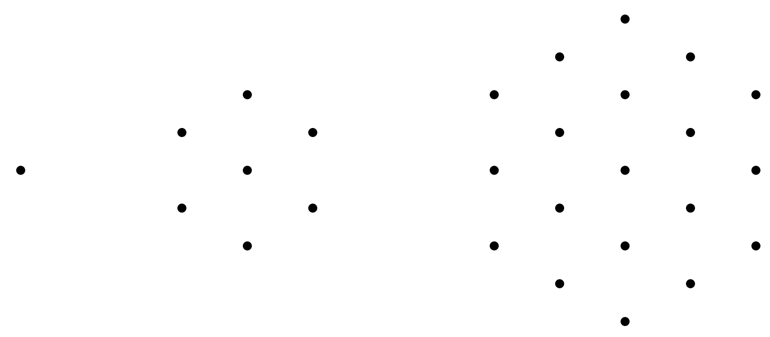
(c) What is the area of the hexagon as a percentage of the area of the circle? [3]

This diagram shows a square inscribed within another circle of radius 1.



(d) What is the area of the square as a percentage of the area of the circle? [5]

10. The centred hexagonal numbers $H_1, H_2, H_3, \ldots$ are so-called because they represent the number of dots needed to make a regular hexagon of varying side-lengths:



$H_1 = 1$ $\quad$ $H_2 = 7$ $\quad$ $H_3 = 19$

(a) With the aid of a diagram, determine the value of the fourth centred hexagonal number. [1]

(b) Leonard suggests that the the nth centred hexagonal number is given by $H_n = 3n(n-1)+1$, dœs this agree with the value you found by counting in part (a)? [1]

(c) Assuming that this formula is correct, find H_{10} and H_{20}. [2]

The corresponding pyramidal number U_n is found by adding the first n centred hexagonal numbers, that is, $U_1 = 1$, $U_2 = 1 + 7$, $U_3 = 1 + 7 + 19$, etc.

(d) Write down U_4. [1]

(e) Albert suggests that the following formula may be used to find these numbers: [2]

$$U_n = 6n^2 - 11n + 6$$

Show clearly that this formula correctly gives U_1, U_2, and U_3.

(f) Dœs it correctly give U_4? [1]

(g) Suggest a formula which would correctly give U_1, U_2, U_3, and U_4 and show that it is correct for U_1, U_2, U_3, and U_4. [2]

11. A Pierpont number of the second kind is a number of the form $2^n3^m - 1$ where n and m are whole numbers greater than or equal to zero.

(a) Calculate the Pierpont number for which $n = 2$ and $m = 1$. Is this number prime? [2]

If the number is a prime number then it is known as a Pierpont prime.

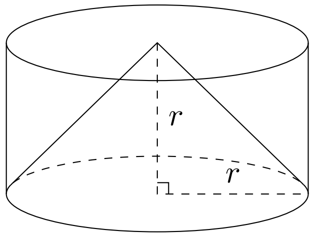
(b) Show that if $n = 5$ and $m = 0$ then the number obtained is a Pierpont prime of the second kind. [2]

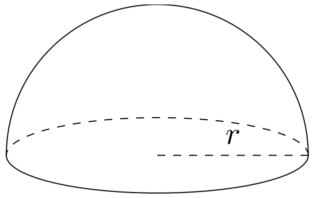
(c) Prove that there is only one Pierpont prime of the second kind for which $n = 0$ and $m > 0$. [3]

(d) What is the smallest prime number which is not a Pierpont prime of the second kind? You must explain how you know this is the smallest number. [1]

(e) What is the largest Pierpont number of the second kind below 100? [2]

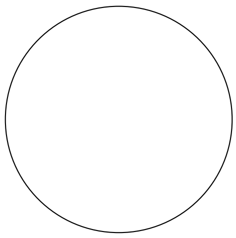
12. The diagram below shows a hemisphere of radius r and a cone contained within a cylinder, both with base radius r and height r.





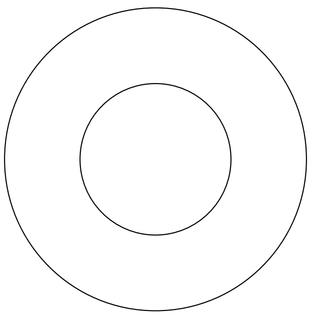
(a) Show that the volume of the hemisphere is the same as the volume of the cylinder with the cone cut out of it. [2]

The top of the sphere is sliced off horizontally h units (where $0 < h < r$) above its circular base. The top of the shape now formed is a circle:



(b) What is the radius of this circle, in terms of h? [2]

Similarly, the top of the other shape is removed, this time k units below its top, to leave an annulus:



(c) What is the radius of the outer circle forming the annulus? [1]

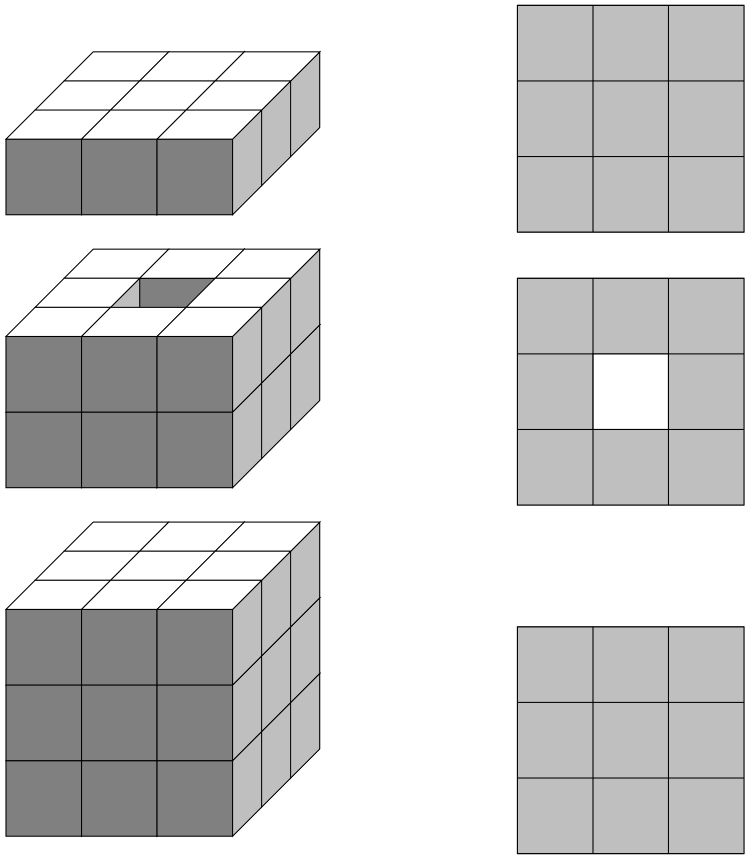
(d) What is the radius of the inner circle, in terms of k? [3]

(e) What relationship must h and k satisfy for the areas of the annulus and the circle from part (b) to be equal? [2]

13. In the final part of this question, marks will be awarded for a good approach to solving the problem as well as for the final answer itself. You are encouraged to show your working and findings clearly.

I have twenty six small cubes, each having side length 1 cm. I glue them together to make a larger, hollow cube. Wherever two faces touch I glue the faces together.

The images below show the structure of the cube, one layer at a time. The left images are a 3D view and the right are top down views of the three layers.



(a) What are the dimensions of the outside of the cube and what are the dimensions of the hollow central void? [1]

(b) State the side length of the cube shaped void when a large hollow cube of outside dimensions n cm is made. (Assume that $n > 2$) [1]

(c) Now give a formula for the number of small cubes needed to make such a hollow cube. [1]

(d) How many pairs of faces would be glued together when I make the large hollow cube using 26 small cubes? [1]

(e) I have a very large supply of such cubes, and can make a solid cube as large as I wish. How many pairs of faces would need to be glued together to form a cube with side length n cm? [6]