Name $\qquad$ Current school


# WELLINGTON <br> COLLEGE 

## I3+ SCHOLARSHIP EXAMINATION 2023

## MATHEMATICS

## TIME ALLOWED: 90 minutes

The marks available for each question are shown in square brackets.

This paper is divided into two sections:
Section A is worth 30 marks and contains seven questions. You should attempt all questions in Section $A$.

Section B is worth 60 marks and contains six questions. You may attempt all questions. Start with the ones that interest you most; answer as many questions as you can. You may find some easier than others.

Write your answers on the question paper.
You may use a calculator.
Credit will be given for the clarity of your work and your explanations.

## Section A (30 marks)

1. Expand and simplify
(a) $-2(8 x-3)+\frac{3}{4}(12 x+16)$
(b) $(x-2)(4+4 x)$
(c) $(3 x-1)^{2}$
(d) $(x+3)^{3}$
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2. Solve
(a) $2 x-4=9$
(b) $4 x+5=6-3 x$
(c) $\frac{x}{3}-\frac{x}{7}=6$
3. (a) Express the ratio $51: 85: 187$ in its simplest form.
(b) There are only green, orange and yellow smarties in a jar, in the ratio $2: 3: 15$. If there are 1200 smarties in total, work out how many of them will be green.
(c) Anaya, Tina and Olivia share money in the ratio $4: 3: 5$. Anaya receives $£ 18$ more than Olivia. Determine the amount that Tina receives.
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4. Factorise fully
(a) $9 x-27$
(b) $k x p^{2}-k x^{2} p$
(c) $x^{4} y^{2}-x^{2} y^{4}$
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5. Make $x$ the subject of
(a) $y=9 x-8$
(b) $4 x-3 y-5=\frac{x+y}{3}$
(c) $\frac{5}{x-y}=x+4$
6. Answer each question
(a) Increase 90 by $2 \%$ [1]
(b) Decrease 810 by $6 \%$
(c) A t-shirt is sold for $£ 22$ on sale. It is marked as " $25 \%$ off". What was the original price of the $t$-shirt?
7. (a) A regular hexagon has a perimeter of 126 cm . What is the length of one side?
(b) $A B C D$ is a square. The shaded area is $10 \mathrm{~cm}^{2}$. What is the length of the square?


## Section B ( 60 marks)

8. A square $A B C D$ of side 12 cm is drawn and two circular arcs are drawn inside it, one centred on $A$ and passing from $B$ to $D$ and another centred on $D$ and passing from $A$ to $C$. The area below the arcs is shaded, as illustrated.


The arcs meet at $E$.
(a) Write down the length of the straight line $A E$.
(b) Hence, explaining your reasoning, give the size of angle $E A D$.
(c) Leaving your answer as a multiple of $\pi$, give the area of the sector $A B E$ (i.e. the area bounded by the straight lines $A B$ and $A E$ and the arc $B E$.
(d) Find the length $E F$, leaving your answer in the form $\sqrt{n}$, where $n$ is a constant to be found.
(e) Hence find, in an exact form, the area of the square which remains unshaded.
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9. In a sequence of isosceles triangles, $A B=B C=C D=D E=\ldots$ as illustrated:

(a) Given that angle $B A C$ is $16^{\circ}$, how many such isosceles triangles can be drawn in this way?
(b) Instead, the initial angle $B A C$ is chosen so that exactly nine triangles can be drawn. Determine, explaining your reasoning, the range of possible values of angle $B A C$.

10. Six towns, no three in a straight line, are connected to each other by straight roads so that in every case there is a direct route between each town, not passing through any other town.
(a) How many roads are necessary to connect the six towns?
(b) State a general result for the number of roads needed for $n$ towns.
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11. Which of the following triangles are right-angled triangles?

## In each case, make your reasoning clear.

(a) A triangle with angles $4 x, 5 x$ and $6 x$ degrees.
(b) A triangle with angles of $y, 3 y$ and $4 y$ degrees.
(c) A triangle with sides $8 \mathrm{~cm}, 9 \mathrm{~cm}$ and 12 cm .
(d) A triangle BED, where ABCDEF is a regular hexagon.
(e) A triangle PQR where M , the mid-point of QR is equidistant from $\mathrm{P}, \mathrm{Q}$ and R .
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12. Inigo and Ed go out running. Each runs at $x \mathrm{~km} / \mathrm{h}$ and jogs at $y \mathrm{~km} / \mathrm{h}$. Inigo takes the same total time to run 6 km and jog 4 km as Ed does to run 3 km and jog 6 km .
(a) Use the formula distance $=\frac{\text { speed }}{\text { time }}$ to make an equation, simplify it and show that $y=\frac{2 x}{3}$.
(b) They now run in opposite directions around a circular track. This track has the centre O.
Both boys start at the same point $P$.
Inigo runs at $x \mathrm{~km} / \mathrm{h}$ and Ed jogs at $y \mathrm{~km} / \mathrm{h}$.
They meet at point G.
Use a diagram to help you calculate the angle POG.

13. A palindromic number is one that reads the same backwards as it does forwards. For example, 1441 and 5831385 are examples of palindromic numbers.
(a) Find a 3-digit palindromic number that is also a square number.
(b) Find a 3-digit palindromic number that is also a prime number.
(c) Find the difference between the largest 5-digit palindromic number and the smallest 6-digit palindromic number.
(d) The 7-digit palindromic number 42 A7A24 is a multiple of 9 . Find the value of the $\operatorname{digit} A$, showing all of your working clearly.
(e) Show that if a four-digit number is palindromic then it is a multiple of 11 .
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